$$z=\frac{(\cos\theta-i\sin\theta)+i(\cos\theta-i\sin\theta)}{\cos\theta}=\frac{(1+i)(\cos\theta-i\sin\theta)}{\cos\theta}$$

or $1+i=[\sqrt{2},\frac{\pi}{4}]$ et $\cos\theta-i\sin\theta=[1,-\theta]$

$$\Rightarrow (1+i)(\cos\theta-i\sin\theta)=[\sqrt{2},\frac{\pi}{4}-\theta]$$

or $\cos\theta>0$ car $\theta\in]\frac{-\pi}{2},\frac{\pi}{2}[$ d'où $z=\frac{\sqrt{2}}{\cos\theta}(\cos(\frac{\pi}{4}-\theta)+i\sin(\frac{\pi}{4}-\theta))$.

5°/ $z=1+\cos\varphi+i\sin\varphi=1+2\cos^2\frac{\varphi}{2}-1+i\cdot 2\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}$

$$=2\cos\frac{\varphi}{2}(\cos\frac{\varphi}{2}+i\sin\frac{\varphi}{2})$$

φ	0		π		2π
$\cos(\frac{\varphi}{2})$		+	0	−	

- Si $\varphi\in]0,\pi[$ alors $z=2\cos\frac{\varphi}{2}(\cos\frac{\varphi}{2}+i\sin\frac{\varphi}{2})$
- Si $\varphi\in]\pi,2\pi[$ alors $z=-2\cos\frac{\varphi}{2}(\cos(\frac{\varphi}{2}+\pi)+i\sin(\frac{\varphi}{2}+\pi))$

6°/ $z=\cos\alpha+i(1+\sin\alpha)=(\cos\alpha+i\sin\alpha)+i$

$$=(\cos\alpha+i\sin\alpha)+(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2})=(\cos\alpha+\cos\frac{\pi}{2})+i(\sin\alpha+\sin\frac{\pi}{2})$$

or $\cos X+\cos Y=2\cos(\frac{X+Y}{2})\cos(\frac{X-Y}{2})$

$\sin X+\sin Y=2\sin(\frac{X+Y}{2})\cos(\frac{X-Y}{2})$

$$\Rightarrow z=2\cos(\frac{\alpha}{2}+\frac{\pi}{4})\cos(\frac{\alpha}{2}-\frac{\pi}{4})+2i\sin(\frac{\alpha}{2}+\frac{\pi}{4})\cos(\frac{\alpha}{2}-\frac{\pi}{4})$$

$$=2\cos(\frac{\alpha}{2}-\frac{\pi}{4})\left[\cos(\frac{\alpha}{2}+\frac{\pi}{4})+i\sin(\frac{\alpha}{2}+\frac{\pi}{4})\right]$$

<u>signe de :</u> $\cos(\frac{\alpha}{2}-\frac{\pi}{4})$ pour $\alpha\in]0,\pi[$:

$$0<\alpha<\pi\Rightarrow 0<\frac{\alpha}{2}<\frac{\pi}{2}\Rightarrow\frac{-\pi}{4}<\frac{\alpha}{2}-\frac{\pi}{4}<\frac{\pi}{4}\Rightarrow\cos(\frac{\alpha}{2}-\frac{\pi}{4})>0$$

d'où $z=2\cos(\frac{\alpha}{2}-\frac{\pi}{4})(\cos(\frac{\alpha}{2}+\frac{\pi}{4})+i\sin(\frac{\alpha}{2}+\frac{\pi}{4}))$